Appendix A
List of Distributions

Here we list common statistical distributions used throughout the book. The often used indicator symbol $1_{\{.\}}$ and gamma function $\Gamma(\alpha)$ are defined as follows.

Definition A.1 The indicator symbol is defined as

$$1_{\{.\}} = \begin{cases} 1, & \text{if condition in } \{.\} \text{ is true}, \\ 0, & \text{otherwise}. \end{cases} \tag{A.1}$$

Definition A.2 The standard gamma function is defined as

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt, \; \alpha > 0. \tag{A.2}$$

A.1 Discrete Distributions

A.1.1 Poisson Distribution, $Poisson(\lambda)$

A Poisson distribution function is denoted as $Poisson(\lambda)$. The random variable N has a Poisson distribution, denoted $N \sim Poisson(\lambda)$, if its probability mass function is

$$p(k) = \Pr[N = k] = \frac{\lambda^k}{k!} e^{-\lambda}, \; \lambda > 0 \tag{A.3}$$

for all $k \in \{0, 1, 2, \ldots\}$. Expectation, variance and variational coefficient of a random variable $N \sim Poisson(\lambda)$ are

$$\mathrm{E}[N] = \lambda, \; \mathrm{Var}[N] = \lambda, \; \mathrm{Vco}[N] = \frac{1}{\sqrt{\lambda}}. \tag{A.4}$$

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A.1.2 Binomial Distribution, $Bin(n, p)$

The binomial distribution function is denoted as $Bin(n, p)$. The random variable N has a binomial distribution, denoted $N \sim Bin(n, p)$, if its probability mass function is

$$p(k) = \Pr[N = k] = \binom{n}{k} p^k (1-p)^{n-k}, \;\; p \in (0, 1), \;\; n \in 1, 2, \ldots \tag{A.5}$$

for all $k \in \{0, 1, 2, \ldots, n\}$. Expectation, variance and variational coefficient of a random variable $N \sim Bin(n, p)$ are

$$\mathrm{E}[N] = np, \;\; \mathrm{Var}[N] = np(1-p), \;\; \mathrm{Vco}[N] = \sqrt{\frac{1-p}{np}}. \tag{A.6}$$

Remark A.1 N is the number of successes in n independent trials, where p is the probability of a success in each trial.

A.1.3 Negative Binomial Distribution, $NegBin(r, p)$

A negative binomial distribution function is denoted as $NegBin(r, p)$. The random variable N has a negative binomial distribution, denoted $N \sim NegBin(r, p)$, if its probability mass function is

$$p(k) = \Pr[N = k] = \binom{r+k-1}{k} p^r (1-p)^k, \;\; p \in (0, 1), \;\; r \in (0, \infty) \tag{A.7}$$

for all $k \in \{0, 1, 2, \ldots\}$. Here, the generalised binomial coefficient is

$$\binom{r+k-1}{k} = \frac{\Gamma(k+r)}{k!\Gamma(r)}, \tag{A.8}$$

where $\Gamma(r)$ is the gamma function.

Expectation, variance and variational coefficient of a random variable $N \sim NegBin(r, p)$ are

$$\mathrm{E}[N] = \frac{r(1-p)}{p}, \mathrm{Var}[N] = \frac{r(1-p)}{p^2}, \;\; \mathrm{Vco}[N] = \frac{1}{\sqrt{r(1-p)}}. \tag{A.9}$$

Remark A.2 If r is a positive integer, N is the number of failures in a sequence of independent trials until r successes, where p is the probability of a success in each trial.

A.2 Continuous Distributions

A.2.1 Uniform Distribution, $\mathcal{U}(a, b)$

A uniform distribution function is denoted as $\mathcal{U}(a, b)$. The random variable X has a uniform distribution, denoted $X \sim \mathcal{U}(a, b)$, if its probability density function is

$$f(x) = \frac{1}{b-a}, \ \ a < b \tag{A.10}$$

for $x \in [a, b]$. Expectation, variance and variational coefficient of a random variable $X \sim \mathcal{U}(a, b)$ are

$$\mathrm{E}[X] = \frac{a+b}{2}, \ \mathrm{Var}[X] = \frac{(b-a)^2}{12}, \ \ \mathrm{Vco}[X] = \frac{b-a}{\sqrt{3}(a+b)}. \tag{A.11}$$

A.2.2 Normal (Gaussian) Distribution, $\mathcal{N}(\mu, \sigma)$

A normal (Gaussian) distribution function is denoted as $\mathcal{N}(\mu, \sigma)$. The random variable X has a normal distribution, denoted $X \sim \mathcal{N}(\mu, \sigma)$, if its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \ \ \sigma^2 > 0, \ \ \mu \in \mathbb{R} \tag{A.12}$$

for all $x \in \mathbb{R}$. Expectation, variance and variational coefficient of a random variable $X \sim \mathcal{N}(\mu, \sigma)$ are

$$\mathrm{E}[X] = \mu, \ \ \mathrm{Var}[X] = \sigma^2, \ \ \mathrm{Vco}[X] = \sigma/\mu. \tag{A.13}$$

A.2.3 Lognormal Distribution, $\mathcal{L}N(\mu, \sigma)$

A lognormal distribution function is denoted as $\mathcal{L}N(\mu, \sigma)$. The random variable X has a lognormal distribution, denoted $X \sim \mathcal{LN}(\mu, \sigma)$, if its probability density function is

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), \ \ \sigma^2 > 0, \ \ \mu \in \mathbb{R} \tag{A.14}$$

for $x > 0$. Expectation, variance and variational coefficient of a random variable $X \sim \mathcal{LN}(\mu, \sigma)$ are

$$\mathrm{E}[X] = e^{\mu+\frac{1}{2}\sigma^2}, \ \mathrm{Var}[X] = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1), \ \mathrm{Vco}[X] = \sqrt{e^{\sigma^2} - 1}. \tag{A.15}$$

A.2.4 t Distribution, $\mathcal{T}(\nu, \mu, \sigma^2)$

A t distribution function is denoted as $\mathcal{T}(\nu, \mu, \sigma^2)$. The random variable X has a t distribution, denoted $X \sim \mathcal{T}(\nu, \mu, \sigma^2)$, if its probability density function is

$$f(x) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)} \frac{1}{\sqrt{\nu\pi}} \left(1 + \frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-(\nu+1)/2} \tag{A.16}$$

for $\sigma^2 > 0$, $\mu \in \mathbb{R}$, $\nu = 1, 2, \ldots$ and all $x \in \mathbb{R}$. Expectation, variance and variational coefficient of a random variable $X \sim \mathcal{T}(\nu, \mu, \sigma^2)$ are

$$\begin{aligned} \mathrm{E}[X] &= \mu \text{ if } \nu > 1, \\ \mathrm{Var}[X] &= \sigma^2 \frac{\nu}{\nu - 2} \text{ if } \nu > 2, \\ \mathrm{Vco}[X] &= \frac{\sigma}{\mu}\sqrt{\frac{\nu}{\nu-2}} \text{ if } \nu > 2. \end{aligned} \tag{A.17}$$

A.2.5 Gamma Distribution, Gamma(α, β)

A gamma distribution function is denoted as $Gamma(\alpha, \beta)$. The random variable X has a gamma distribution, denoted as $X \sim Gamma(\alpha, \beta)$, if its probability density function is

$$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha} \exp(-x/\beta), \ \alpha > 0, \ \beta > 0 \tag{A.18}$$

for $x > 0$. Expectation, variance and variational coefficient of a random variable $X \sim Gamma(\alpha, \beta)$ are

$$\mathrm{E}[X] = \alpha\beta, \ \mathrm{Var}[X] = \alpha\beta^2, \ \mathrm{Vco}[X] = 1/\sqrt{\alpha}. \tag{A.19}$$

A.2.6 Weibull Distribution, Weibull(α, β)

A Weibull distribution function is denoted as $Weibull(\alpha, \beta)$. The random variable X has a Weibull distribution, denoted as $X \sim Weibull(\alpha, \beta)$, if its probability density function is

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \exp(-(x/\beta)^\alpha), \ \alpha > 0, \ \beta > 0 \tag{A.20}$$

for $x > 0$. The corresponding distribution function is

$$F(x) = 1 - \exp\left(-(x/\beta)^{\alpha}\right), \ \alpha > 0, \ \beta > 0. \tag{A.21}$$

Expectation and variance of a random variable $X \sim Weibull(\alpha, \beta)$ are

$$\mathrm{E}[X] = \beta\Gamma(1 + 1/\alpha), \ \mathrm{Var}[X] = \beta^2 \left(\Gamma(1 + 2/\alpha) - (\Gamma(1 + 1/\alpha))^2\right).$$

A.2.7 Pareto Distribution (One-Parameter), $Pareto(\xi, x_0)$

A one-parameter Pareto distribution function is denoted as $Pareto(\xi, x_0)$. The random variable X has a Pareto distribution, denoted as $X \sim Pareto(\xi, x_0)$, if its distribution function is

$$F(x) = 1 - \left(\frac{x}{x_0}\right)^{-\xi}, \quad x \geq x_0, \tag{A.22}$$

where $x_0 > 0$ and $\xi > 0$. The support starts at x_0, which is typically known and not considered as a parameter. Therefore the distribution is referred to as a single parameter Pareto. The corresponding probability density function is

$$f(x) = \frac{\xi}{x_0}\left(\frac{x}{x_0}\right)^{-\xi-1}. \tag{A.23}$$

Expectation, variance and variational coefficient of $X \sim Pareto(\xi, x_0)$ are

$$\begin{aligned}
\mathrm{E}[X] &= x_0\frac{\xi}{\xi - 1} \ \text{ if } \ \xi > 1, \\
\mathrm{Var}[X^2] &= x_0^2\frac{\xi}{(\xi - 1)^2(\xi - 2)} \ \text{ if } \ \xi > 2, \\
\mathrm{Vco}[X] &= \frac{1}{\sqrt{\xi(\xi - 2)}} \ \text{ if } \ \xi > 2.
\end{aligned}$$

A.2.8 Pareto Distribution (Two-Parameter), $Pareto_2(\alpha, \beta)$

A two-parameter Pareto distribution function is denoted as $Pareto_2(\alpha, \beta)$. The random variable X has a Pareto distribution, denoted as $X \sim Pareto_2(\alpha, \beta)$, if its distribution function is

$$F(x) = 1 - \left(1 + \frac{x}{\beta}\right)^{-\alpha}, \quad x \geq 0, \tag{A.24}$$

where $\alpha > 0$ and $\beta > 0$. The corresponding probability density function is

$$f(x) = \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}}. \tag{A.25}$$

The moments of a random variable $X \sim Pareto_2(\alpha, \beta)$ are

$$\mathrm{E}[X^k] = \frac{\beta^k k!}{\prod_{i=1}^{k}(\alpha - i)}; \quad \alpha > k.$$

A.2.9 Generalised Pareto Distribution, GPD(ξ, β)

A GPD distribution function is denoted as $GPD(\xi, \beta)$. The random variable X has a GPD distribution, denoted as $X \sim GPD(\xi, \beta)$, if its distribution function is

$$H_{\xi,\beta}(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi}, \ \xi \neq 0, \\ 1-\exp(-x/\beta), \ \xi = 0, \end{cases} \tag{A.26}$$

where $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$. The corresponding probability density function is

$$h(x) = \begin{cases} \frac{1}{\beta}(1+\xi x/\beta)^{-\frac{1}{\xi}-1}, \ \xi \neq 0, \\ \frac{1}{\beta}\exp(-x/\beta), \ \xi = 0. \end{cases} \tag{A.27}$$

Expectation, variance and variational coefficient of $X \sim GPD(\xi, \beta)$, $\xi \geq 0$, are

$$\begin{aligned} \mathrm{E}[X^n] &= \frac{\beta^n n!}{\prod_{k=1}^{n}(1-k\xi)}, \ \xi < \frac{1}{n}; \ \mathrm{E}[X] = \frac{\beta}{1-\xi}, \ \xi < 1; \\ \mathrm{Var}[X^2] &= \frac{\beta^2}{(1-\xi)^2(1-2\xi)}, \ \mathrm{Vco}[X] = \frac{1}{\sqrt{1-2\xi}}, \ \xi < \frac{1}{2}. \end{aligned} \tag{A.28}$$

A.2.10 Beta Distribution, Beta(α, β)

A beta distribution function is denoted as $Beta(\alpha, \beta)$. The random variable X has a beta distribution, denoted as $X \sim Beta(\alpha, \beta)$, if its probability density function is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \leq x \leq 1, \tag{A.29}$$

for $\alpha > 0$ and $\beta > 0$. Expectation, variance and variational coefficient of a random variable $X \sim Beta(\alpha, \beta)$ are

$$\mathrm{E}[X] = \frac{\alpha}{\alpha+\beta}, \; \mathrm{Var}[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(1+\alpha+\beta)}, \; \mathrm{Vco}[X] = \sqrt{\frac{\beta}{\alpha(1+\alpha+\beta)}}.$$

A.2.11 Generalised Inverse Gaussian Distribution, GIG(ω, ϕ, ν)

A GIG distribution function is denoted as $GIG(\omega, \phi, \nu)$. The random variable X has a GIG distribution, denoted as $X \sim GIG(\omega, \phi, \nu)$, if its probability density function is

$$f(x) = \frac{(\omega/\phi)^{(\nu+1)/2}}{2K_{\nu+1}\left(2\sqrt{\omega\phi}\right)} x^{\nu} e^{-x\omega - x^{-1}\phi}, \quad x > 0, \tag{A.30}$$

where $\phi > 0, \omega \geq 0$ if $\nu < -1$; $\phi > 0, \omega > 0$ if $\nu = -1$; $\phi \geq 0, \omega > 0$ if $\nu > -1$; and

$$K_{\nu+1}(z) = \frac{1}{2}\int_0^{\infty} u^{\nu} e^{-z(u+1/u)/2} \mathrm{d}u.$$

$K_\nu(z)$ is called a modified Bessel function of the third kind; see for instance Abramowitz and Stegun ([3], p. 375).

The moments of a random variable $X \sim GIG(\omega, \phi, \nu)$ are not available in a closed form through elementary functions but can be expressed in terms of Bessel functions:

$$\mathrm{E}[X^\alpha] = \left(\frac{\phi}{\omega}\right)^{\alpha/2} \frac{K_{\nu+1+\alpha}\left(2\sqrt{\omega\phi}\right)}{K_{\nu+1}\left(2\sqrt{\omega\phi}\right)}, \quad \alpha \geq 1, \; \phi > 0, \; \omega > 0.$$

Often, using notation $R_\nu(z) = K_{\nu+1}(z)/K_\nu(z)$, it is written as

$$\mathrm{E}[X^\alpha] = \left(\frac{\phi}{\omega}\right)^{\alpha/2} \prod_{k=1}^{\alpha} R_{\nu+k}\left(2\sqrt{\omega\phi}\right), \quad \alpha = 1, 2, \ldots$$

The mode is easily calculated from $\frac{\partial}{\partial x} x^\nu e^{-(\omega x + \phi/x)} = 0$ as

$$\mathrm{mode}[X] = \frac{1}{2\omega}\left(\nu + \sqrt{\nu^2 + 4\omega\phi}\right),$$

that differs only slightly from the expected value for large ν, i.e.

$$\mathrm{mode}[X] \to \mathrm{E}[X] \qquad \text{for} \quad \nu \to \infty.$$

A.2.12 *d*-variate Normal Distribution, $\mathcal{N}_d(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

A d-variate normal distribution function is denoted as $\mathcal{N}_d(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = (\mu_1, \ldots \mu_d)' \in \mathbb{R}^d$ and $\boldsymbol{\Sigma}$ is a positive definite matrix ($d \times d$). The corresponding probability density function is

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}\sqrt{\det \boldsymbol{\Sigma}}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right), \ \mathbf{x} \in \mathbb{R}^d, \tag{A.31}$$

where $\boldsymbol{\Sigma}^{-1}$ is the inverse of the matrix $\boldsymbol{\Sigma}$. Expectations and covariances of a random vector $\mathbf{X} = (X_1, \ldots, X_d)' \sim \mathcal{N}_d(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ are

$$\mathrm{E}[X_i] = \mu_i, \ \mathrm{Cov}[X_i, X_j] = \Sigma_{i,j}, \ i, j = 1, \ldots, d. \tag{A.32}$$

A.2.13 *d*-variate *t*-Distribution, $\mathcal{T}_d(\nu, \boldsymbol{\mu}, \boldsymbol{\Sigma})$

A d-variate t-distribution function with ν degrees of freedom is denoted as $\mathcal{T}_d(\nu, \boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\nu > 0$, $\boldsymbol{\mu} = (\mu_1, \ldots \mu_d)' \in \mathbb{R}^d$ is a location vector and $\boldsymbol{\Sigma}$ is a positive definite matrix ($d \times d$). The corresponding probability density function is

$$f(\mathbf{x}) = \frac{\Gamma\left(\frac{\nu+d}{2}\right)}{(\nu\pi)^{d/2}\Gamma\left(\frac{\nu}{2}\right)\sqrt{\det \boldsymbol{\Sigma}}} \left(1 + \frac{(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}{\nu}\right)^{-\frac{\nu+d}{2}}, \tag{A.33}$$

where $\mathbf{x} \in \mathbb{R}^d$ and $\boldsymbol{\Sigma}^{-1}$ is the inverse of the matrix $\boldsymbol{\Sigma}$. Expectations and covariances of a random vector $\mathbf{X} = (X_1, \ldots, X_d)' \sim \mathcal{T}_d(\nu, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ are

$$\begin{aligned} \mathrm{E}[X_i] &= \mu_i, \ \text{if } \nu > 1, \ i = 1, \ldots, d; \\ \mathrm{Cov}[X_i, X_j] &= \nu\Sigma_{i,j}/(\nu-2), \ \text{if } \nu > 2, \ i, j = 1, \ldots, d. \end{aligned} \tag{A.34}$$

Appendix B
Selected Simulation Algorithms

B.1 Simulation from GIG Distribution

To generate realisations of a random variable $X \sim \text{GIG}(\omega, \phi, \nu)$ with $\omega, \phi > 0$, a special algorithm is required because we cannot invert the distribution function in closed form. The following algorithm can be found in Dagpunar [67]:

Algorithm B.1 (Simulation from GIG distribution)

1. $\alpha = \sqrt{\omega/\phi}$; $\beta = 2\sqrt{\omega\phi}$,
   $m = \frac{1}{\beta}\left(\nu + \sqrt{\nu^2 + \beta^2}\right)$,
   $g(y) = \frac{1}{2}\beta y^3 - y^2\left(\frac{1}{2}\beta m + \nu + 2\right) + y\left(\nu m - \frac{\beta}{2}\right) + \frac{1}{2}\beta m.$
2. Set $y_0 = m$,
   While $g(y_0) \leq 0$ do $y_0 = 2y_0$,
   y_+: root of g in the interval (m, y_0),
   y_-: root of g in the interval $(0, m)$.
3. $a = (y_+ - m)\left(\frac{y_+}{m}\right)^{\nu/2} \exp\left(-\frac{\beta}{4}\left(y_+ + \frac{1}{y_+} - m - \frac{1}{m}\right)\right)$,
   $b = (y_- - m)\left(\frac{y_-}{m}\right)^{\nu/2} \exp\left(-\frac{\beta}{4}\left(y_- + \frac{1}{y_-} - m - \frac{1}{m}\right)\right)$,
   $c = -\frac{\beta}{4}\left(m + \frac{1}{m}\right) + \frac{\nu}{2}\ln(m)$.
4. Repeat $U, V \sim \mathcal{U}(0, 1)$, $Y = m + a\frac{U}{V} + b\frac{1-V}{U}$,
   until $Y > 0$ and $-\ln U \geq -\frac{\nu}{2}\ln Y + \frac{1}{4}\beta\left(Y + \frac{1}{Y}\right) + c$,
   Then $X = \frac{Y}{\alpha}$ is $\text{GIG}(\omega, \phi, \nu)$.

To generate a sequence of n realisations from a GIG random variable, step 4 is repeated n times.

B.2 Simulation from α-stable Distribution

To generate realisations of a random variable $X \sim \alpha\text{Stable}(\alpha, \beta, \sigma, \mu)$, defined by (6.56), a special algorithm is required because the density of α-stable distribution is not available in closed form. An elegant and efficient solution was proposed in Chambers, Mallows and Stuck [50]; also see Nolan [176].

Algorithm B.2 (Simulation from α-stable distribution)

1. Simulate W from the exponential distribution with mean = 1.
2. Simulate U from $\mathcal{U}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
3. Calculate

$$Z = \begin{cases} S_{\alpha,\beta} \frac{\sin(\alpha(U+B_{\alpha,\beta}))}{(\cos U)^{1/\alpha}} \left(\frac{\cos(U-\alpha(U+B_{\alpha,\beta}))}{W}\right)^{-1+\frac{1}{\alpha}}, & \alpha \neq 1, \\ \frac{2}{\pi}\left(\left(\frac{\pi}{2} + \beta U\right)\tan U - \beta \ln\left(\frac{\pi W \cos U}{\pi + 2\beta U}\right)\right), & \alpha = 1, \end{cases}$$

   where

$$S_{\alpha,\beta} = (1 + \beta^2 \tan^2(\pi\alpha/2))^{\frac{1}{2\alpha}},$$
$$B_{\alpha,\beta} = \frac{1}{\alpha} \arctan(\beta \tan(\pi\alpha/2)).$$

   The obtained Z is a sample from $\alpha\text{Stable}(\alpha, \beta, 1, 0)$.
4. Then,

$$X = \begin{cases} \mu + \sigma Z, & \alpha \neq 1, \\ \mu + \sigma Z + \frac{2}{\pi}\beta\sigma \ln \sigma, & \alpha = 1, \end{cases}$$

   is a sample from $\alpha\text{Stable}(\alpha, \beta, \sigma, \mu)$.

Note that there are different parameterisations of the α-stable distribution. The algorithm above is for representation (6.56).

Solutions for Selected Problems

Problems of Chapter 2

2.1 The likelihood function for independent data $\mathbf{N} = \{N_1, N_2, \ldots, N_M\}$ from $Poisson(\lambda)$ is

$$\ell_{\mathbf{n}}(\lambda) = \prod_{i=1}^{M} e^{-\lambda} \frac{\lambda^{n_i}}{n_i!},$$

$$\ln \ell_{\mathbf{n}}(\lambda) = -\lambda M + \ln \lambda \sum_{i=1}^{M} n_i - \sum_{i=1}^{M} \ln(n_i!).$$

The MLE $\widehat{\Lambda}$ maximising the log-likelihood function $\ln \ell_{\mathbf{N}}(\lambda)$ is

$$\widehat{\Lambda} = \frac{1}{M} \sum_{i=1}^{M} N_i.$$

Using the properties of the Poisson distribution, $\mathrm{E}[N_i] = \mathrm{Var}[N_i] = \lambda$, it is easy to get

$$\mathrm{E}[\widehat{\Lambda}] = \frac{1}{M} \sum_{i=1}^{M} \mathrm{E}[N_i] = \lambda;$$

$$\mathrm{Var}[\widehat{\Lambda}] = \frac{1}{M^2} \sum_{i=1}^{M} \mathrm{Var}[N_i] = \frac{\lambda}{M}.$$

To estimate the variance of $\widehat{\Lambda}$ using a normal approximation, find the information matrix

$$\mathrm{I}(\lambda) = -\frac{1}{M} \mathrm{E}\left[\frac{\partial^2 \ln \ell_{\mathbf{N}}(\lambda)}{\partial \lambda^2}\right] = \frac{1}{M\lambda^2} \mathrm{E}\left[\sum_{i=1}^{M} N_i\right] = \frac{1}{\lambda}.$$

Thus, using asymptotic normal distribution approximation,

$$\mathrm{Var}[\widehat{\Lambda}] \approx \mathrm{I}^{-1}(\lambda)/M = \lambda/M.$$

In both cases the variance depends on unknown true parameter λ that can be estimated, for a given realisation $\mathbf{n}$, as $\widehat{\lambda}$.

2.4 Consider

$$L(\mathbf{u}) = u_1 L_1 + \cdots + u_J L_J,$$

where $\mathbf{u} \in \mathbb{R}^J$ and set

$$\phi_{\mathbf{u}}(t) = \varrho[tL(\mathbf{u})]], \quad t > 0.$$

Then using homogeneity property $\varrho[tL] = t\varrho[L]$,

$$\frac{d\phi_{\mathbf{u}}(t)}{dt} = \varrho[L(u)].$$

From another side

$$\frac{d\phi_{\mathbf{u}}(t)}{dt} = \sum_{j=1}^{J} \frac{\varrho[L(\mathbf{x})]}{\partial x_j}\bigg|_{\mathbf{x}=t\mathbf{u}} u_j = \sum_{j=1}^{J} \frac{\varrho[L(\mathbf{u})]}{\partial u_j} u_j,$$

where to get the last equality we used homogeneity property. Thus

$$\varrho[L(\mathbf{1})] = \sum_{j=1}^{J} \frac{\varrho[L_1 + \cdots + L_j + hL_j]}{\partial h}\bigg|_{h=0}$$

completes the proof.

2.5 The sum of risks is gamma distributed:

$$Z_1 + Z_2 + Z_3 \sim Gamma(\alpha_1 + \alpha_2 + \alpha_3, \beta).$$

Thus $\mathrm{VaR}_{0.999}[Z_i] = F_G^{-1}(0.999|\alpha_i, \beta)$ and

$$\mathrm{VaR}_{0.999}[Z_1 + Z_2 + Z_3] = F_G^{-1}(0.999|\alpha_1 + \alpha_2 + \alpha_3, \beta),$$

where $F_G^{-1}(\cdot|\alpha, \beta)$ is the inverse of the $Gamma(\alpha, \beta)$. Using, for example, MS Excel spreadsheet function GAMMAINV($\cdot$), find

$$\mathrm{VaR}_{0.999}[Z_1] \approx 5.414, \quad \mathrm{VaR}_{0.999}[Z_2] \approx 6.908,$$
$$\mathrm{VaR}_{0.999}[Z_3] \approx 8.133, \quad \mathrm{VaR}_{0.999}[Z_1 + Z_2 + Z_3] \approx 11.229.$$

The sum of VaRs is $\mathrm{VaR}_{0.999}[Z_1] + \mathrm{VaR}_{0.999}[Z_2] + \mathrm{VaR}_{0.999}[Z_3] \approx 20.455$ and thus the diversification is $\approx 45\%$.

Problems of Chapter 3

3.1 By definition of the expected shortfall we have

$$\mathrm{E}[Z|Z > L] = \frac{1}{1 - H(L)} \int_L^\infty z h(z) dz$$
$$= \frac{\mathrm{E}[Z]}{1 - H(L)} - \frac{1}{1 - H(L)} \int_0^L z h(z) dz.$$

Substituting $h(z)$ calculated via characteristic function (3.11) and changing variable $x = t \times L$, we obtain

$$\int_0^L z h(z) dz = \frac{2}{\pi} \int_0^L z \int_0^\infty \mathrm{Re}[\chi(t)] \cos(tz) dt dz$$
$$= \frac{2L}{\pi} \int_0^\infty \mathrm{Re}[\chi\ (x/L)] \left[\frac{\sin(x)}{x} - \frac{1 - \cos(x)}{x^2} \right] dx.$$

Recognizing that the term involving $\sin(x)/x$ corresponds to $H(L)$, we obtain

$$\mathrm{E}[Z|Z > L] = \frac{1}{1 - H(L)} \left[\mathrm{E}[Z] - H(L)L + \frac{2L}{\pi} \int_0^\infty \mathrm{Re}[\chi(x/L)] \frac{1 - \cos x}{x^2} dx \right].$$

Problems of Chapter 4

4.1 The linear estimator $\widehat{\theta}_{tot} = w_1 \widehat{\theta}_1 + \cdots + w_K \widehat{\theta}_K$ is unbiased, i.e. $\mathrm{E}[\widehat{\theta}_{tot}] = \theta$, if $w_1 + \cdots + w_K = 1$ because $\mathrm{E}[\widehat{\theta}_k] = \theta$. Minimisation of the variance

$$\mathrm{Var}[\widehat{\theta}_{tot}] = w_1^2 \sigma_1^2 + \cdots + w_K^2 \sigma_K^2$$

under the constraint $w_1 + \cdots + w_K$ is equivalent to unconstrained minimisation of the

$$\Psi = \mathrm{Var}[\widehat{\theta}_{tot}] - \lambda(w_1 + \cdots + w_K),$$

which is a well-known method of Lagrange multipliers. Optimisation of the above requires solution of the following equations:

$$\frac{\partial \Psi}{\partial w_i} = 2w_i\sigma_i^2 - \lambda = 0, \quad i = 1, \dots, K;$$
$$\frac{\partial \Psi}{\partial \lambda} = -(w_1 + \cdots + w_K) = 0.$$

That gives

$$\frac{1}{2}\lambda = \left(\sum_{k=1}^{K}\left(1/\sigma_k^2\right)\right)^{-1}, \quad w_i = \frac{1/\sigma_i^2}{\sum_{k=1}^{K}\left(1/\sigma_k^2\right)}.$$

4.2 Given $\Theta = \theta$, the joint density of the data at $\mathbf{N} = \mathbf{n}$ is

$$f(\mathbf{n}|\theta) \propto \prod_{i=1}^{T} \theta^{n_i}(1-\theta)^{V_i - n_i}.$$

From Bayes's theorem, the posterior density of θ is $\pi(\theta|\mathbf{n}) \propto f(\mathbf{n}|\theta)\pi(\theta)$, where $\pi(\theta)$ is the prior density. Thus

$$\pi(\theta|\mathbf{n}) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}\prod_{i=1}^{T}\theta^{n_i}(1-\theta)^{V_i-n_i}$$
$$= \theta^{\alpha_T - 1}(1-\theta)^{\beta_T - 1},$$

where

$$\alpha_T = \alpha + \sum_{i=1}^{T} n_i, \quad \beta_T = \beta + \sum_{i=1}^{T} V_i - \sum_{i=1}^{T} n_i.$$

Thus the posterior distribution of Θ is $Beta(\alpha_T, \beta_T)$.

Problems of Chapter 5

5.1 Denote the data above L as $\widetilde{\mathbf{X}} = (\widetilde{X}_1, \widetilde{X}_2, \dots, \widetilde{X}_k)'$. These random variables are independent with a common density $f(x|\xi)/(1 - F(L|\xi))$, $x \geq L$, where $f(x|\xi)$ is the density of the Pareto distribution $F(x|\xi) = 1 - (x/a)^{-\xi}$, $x \geq a > 0$. Thus the likelihood function for given data above L is

$$\ell_{\widetilde{\mathbf{x}}}(\xi) = \prod_{i=1}^{k} \frac{f(\widetilde{x}_i|\xi)}{1 - F(L|\xi)}.$$

Substituting the Pareto density

$$f(x|\xi) = \frac{\xi}{a}\left(\frac{x}{a}\right)^{-\xi-1}$$

gives

$$\ln \ell_{\widetilde{\mathbf{x}}}(\xi) = K\xi \ln(L/a) + K \ln(\xi/a) - (\xi + 1)\sum_{i=1}^{K} \ln(\widetilde{x}_i/a).$$

Then, solving $\partial \ln \ell_{\widetilde{\mathbf{x}}}(\xi)/\partial\xi = 0$, we obtain

$$\widehat{\xi}^{\text{MLE}} = \left(-\ln(L/a) + \frac{1}{K}\sum_{i=1}^{K} \ln(\widetilde{x}_i/a)\right)^{-1}.$$

Problems of Chapter 6

6.1 The probability generating function of the negative binomial, $NegBin(r, p)$, is $\psi(t) = (1 - (t-1)(1-p)/p)^{-r}$. Then, using formula (6.29), we obtain that the distribution of the maximum loss over one year is

$$F_M(x) = \psi(F(x)) = \left(1 + \frac{1-p}{p}(1 - F(x))\right)^{-r},$$

where $F(x) = 1 - \exp(-x/\beta)$ is the severity distribution. The distribution of the maximum loss over m years is simply

$$(F_M(x))^m = \left(1 + \frac{1-p}{p}(1 - F(x))\right)^{-r\times m}.$$

Problems of Chapter 7

7.1 Consider random variables U_1 and U_2 from the t-copula $C^{(t)}_{\nu,\rho}(u_1, u_2)$. By definition, the lower tail dependence is

$$\lambda_L = \lim_{q\to 0+} \frac{C^{(t)}_{\nu,\rho}(q, q)}{q}.$$

Due to the radial symmetry of the t-copula, the upper tail dependence λ_U is the same as λ_L. Applying L'Hôpital's rule, that is, taking derivatives of the nominator and denominator,

$$\lambda_L = \lim_{q\to 0+} \frac{dC^{(t)}_{\nu,\rho}(q,q)}{dq} = \lim_{q\to 0+} \{\Pr[U_2 \leq q|U_1 = q] + \Pr[U_1 \leq q|U_2 = q]\}.$$

Let $X_1 = F_\nu^{(-1)}(U_1)$ and $X_2 = F_\nu^{(-1)}(U_2)$, where $F_\nu(\cdot)$ is a standard univariate t-distribution with ν degrees of freedom, $\mathcal{T}(\nu, 0, 1)$. Thus $(X_1, X_2)'$ is from a bivariate t-distribution $\mathcal{T}_2(\nu, 0, \mathbf{\Sigma})$, where $\mathbf{\Sigma}$ is a correlation matrix with off-diagonal element ρ. Then, one can calculate the conditional density of X_2 given $X_1 = x_1$:

$$f(x_2|x_1) = \frac{f(x_1,x_2)}{f(x_1)} \propto \left(1 + \frac{\nu+1}{(1-\rho^2)(\nu+x_1^2)} \frac{(x_2-\rho x_1)^2}{\nu+1}\right)^{-(\nu+2)/2}.$$

This can be recognised as a univariate t distribution $\mathcal{T}(\nu+1, \mu, \sigma^2)$ with the mean $\mu = \rho x_1$, $\sigma^2 = \frac{(1-\rho^2)(\nu+x_1^2)}{\nu+1}$ and $\nu + 1$ degrees of freedom. Thus

$$\Pr[X_2 \leq x|X_1 = x] = F_{\nu+1}\left(\frac{(x - x\rho)\sqrt{\nu+1}}{\sqrt{(1-\rho^2)(\nu+x^2)}}\right).$$

Finally, using that $\Pr[X_1 \leq x|X_2 = x] = \Pr[X_2 \leq x|X_1 = x]$ and taking limit $x \to -\infty$ we get

$$\lambda = 2F_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}}\right).$$

References

1. Abate, J., Whitt, W.: Numerical inversion of Laplace transforms of probability distributions. ORSA Journal of Computing **7**, 36–43 (1992)
2. Abate, J., Whitt, W.: Numerical inversion of probability generating functions. Operations Research Letters **12**, 245–251 (1995)
3. Abramowitz, M., Stegun, I.A.: Handbook of Mathematical Functions. Dover Publications, New York, NY (1965)
4. Acerbi, C., Tasche, D.: On the coherence of expected shortfall. Journal of Banking and Finance **26**, 1487–1503 (2002)
5. Akaike, H.: Information measure and model selection. Bulletin of the International Statistical Institute **50**, 277–290 (1983)
6. Alderweireld, T., Garcia, J., Léonard, L.: A practical operational risk scenario analysis quantification. Risk Magazine **19**(2), 93–95 (2006)
7. Ale, B.J.M., Bellamy, L.J., van der Boom, R., Cooper, J., Cooke, R.M., Goossens, L.H.J., Hale, A.R., Kurowicka, D., Morales, O., Roelen, A.L.C., Spouge, J.: Further development of a causal model for air transport safety (CATS); building the mathematical heart. Reliability Engineering and System Safety **94**(9), 1433–1441 (2009)
8. Allen, L., Bali, T.: Cyclicality in catastrophic and operational risk measurements. Baruch College (2004). Technical report
9. Allen, L., Boudoukh, J., Saunders, A.: Understanding Market, Credit and Operational Risk: The Value-at-Risk Approach. Blackwell Publishing, Oxford (2005)
10. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Mathematical Finance **9**, 203–228 (1999)
11. Atchade, Y., Rosenthal, J.: On adaptive Markov chain Monte Carlo algorithms. Bernoulli **11**(5), 815–828 (2005)
12. Aue, F., Klakbrener, M.: LDA at work: Deutsche Bank's approach to quantify operational risk. The Journal of Operational Risk **1**(4), 49–95 (2006)
13. Basel Committee on Banking Supervision: Operational Risk Management. Bank for International Settlements (September 1998). URL www.bis.org
14. Basel Committee on Banking Supervision: Quantitative Impact Study for Operational Risk: Overview of Individual Loss Data and Lessons Learned. Bank for International Settlements (January 2002). URL www.bis.org/bcbs/qis/qishist.htm
15. Basel Committee on Banking Supervision: The 2002 Loss Data Collection Exercise for Operational Risk: Summary of the Data Collected. Bank for International Settlements (March 2003). URL www.bis.org/bcbs/qis/ldce2002.htm
16. Basel Committee on Banking Supervision: International Convergence of Capital Measurement and Capital Standards: a revised framework. Bank for International Settlements, Basel (June 2004). URL www.bis.org
17. Basel Committee on Banking Supervision: International Convergence of Capital Measurement and Capital Standards: a revised framework. Bank for International Settlements, Basel (June 2006). URL www.bis.org

18. Basel Committee on Banking Supervision: Results from the 2008 Loss Data Collection Exercise for Operational Risk. Bank for International Settlements (July 2009). URL www.bis.org
19. Basel Committee on Banking Supervision: Working Paper on the Regulatory Treatment of Operational Risk. Bank for International Settlements (September 2001). URL www.bis.org
20. Baud, N., Frachot, A., Roncalli, T.: How to avoid over-estimating capital charge for operational risk? OpRisk&Compliance (February 2003)
21. Bayes, T.: An essay towards solving a problem in the doctrine of chances. Philosophical Transactions of the Royal Society of London **53**, 370–418 (1763)
22. Bazzarello, D., Crielaard, B., Piacenza, F., Soprano, A.: Modeling insurance mitigation on operational risk capital. The Journal of Operational Risk **1**(1), 57–65 (2006)
23. Beaumont, M.A., Zhang, W., Balding, D.J.: Approximate Bayesian computation in population genetics. Genetics **162**, 2025–2035 (2002)
24. Bedard, M., Rosenthal, J.S.: Optimal scaling of Metropolis algorithms: heading towards general target distributions. The Canadian Journal of Statistics **36**(4), 483–503 (2008)
25. Bee, M.: Copula-based multivariate models with applications to risk management and insurance. Dipartimento di Economia, Universita degli Studi di Trento (2005). URL www.gloriamundi.org. Working paper
26. Bee, M.: On Maximum Likelihood Estimation of Operational Loss Distributions. Dipartimento di Economia, Universita degli Studi di Trento (2005). Discussion paper No.3
27. Berger, J.O.: Statistical Decision Theory and Bayesian Analysis, 2nd edn. Springer, New York, NY (1985)
28. Bladt, M.: A review of phase-type distributions and their use in risk theory. ASTIN Bulletin **35**(1), 145–167 (2005)
29. Böcker, K., Klüppelberg, C.: Operational VAR: a closed-form approximation. Risk Magazine **12**, 90–93 (2005)
30. Böcker, K., Klüppelberg, C.: Modelling and measuring multivariate operational risk with Lévy copulas. The Journal of Operational Risk **3**(2), 3–27 (2008)
31. Böcker, K., Klüppelberg, C.: Multivariate models for Operational Risk. Quantitative Finance **10**(8), 855–869 (2010)
32. Böcker, K., Sprittulla, J.: Operational VAR: meaningful means. Risk Magazine **12**, 96–98 (2006)
33. Bohman, H.: Numerical inversion of characteristic functions. Scandinavian Actuarial Journal pp. 121–124 (1975)
34. Bookstabber, R.M., McDonald, J.B.: A general distribution for describing security price returns. The Journal of Business **60**(3), 401–424 (1987)
35. Brass, H., Förster, K.J.: On the estimation of linear functionals. Analysis **7**, 237–258 (1987)
36. Brewer, M.J., Aitken, C.G.G., Talbot, M.: A comparison of hybrid strategies for Gibbs sampling in mixed graphical models. Computational Statistics and Data Analysis **21**(3), 343–365 (1996)
37. Brigham, E.O.: The Fast Fourier Transform. Prentice-Hall, Englewood Cliffs, NJ (1974)
38. van den Brink, J.: Operational Risk: The New Challenge for Banks. Pulgrave, London (2002)
39. British Bankers Association: Operational Risk Management Survey (December 1999). URL www.bba.org.uk
40. Buch-Kromann, T.: Comparison of tail performance of the Champernowne transformed kernel density estimator, the generalized Pareto distribution and the g-and-h distribution. The Journal of Operational Risk **4**(2), 43–67 (2009)
41. Buch-Larsen, T., Nielsen, J.P., Guillen, M., Bolance, C.: Kernel density estimation for heavy-tailed distributions using the Champernowne transformation. Statistics **39**(6), 503–518 (2005)
42. Bühlmann, H.: Mathematical Methods in Risk Theory. Springer, New York, NY (1970)
43. Bühlmann, H.: Numerical evaluation of the compound Poisson distribution: recursion or Fast Fourier Transform? Scandinavian Actuarial Journal **2**, 116–126 (1984)
44. Bühlmann, H., Gisler, A.: A Course in Credibility Theory and its Applications. Springer, Berlin (2005)

45. Bühlmann, H., Shevchenko, P.V., Wüthrich, M.V.: A "toy" model for operational risk quantification using credibility theory. The Journal of Operational Risk **2**(1), 3–19 (2007)
46. Bühlmann, H., Straub, E.: Glaubwürdigkeit für schadensätze. Bulletin of the Swiss Association of Actuaries **70**, 111–133 (1970)
47. Burnecki, K., Kukla, G., Taylor, D.: Pricing of catastrophic bonds. In: P. Cízek, W. Härdle, R. Weron (eds.) Statistical Tools for Finance and Insurance. Springer, New York, NY (2005)
48. Carter, M., Van Brunt, B.: The Lebesgue-Stieltjes Integral. A Practical Introduction. Springer, New York, NY (2000)
49. Casella, G., Berger, R.L.: Statistical Inference. Duxbury, Pasific Grove (2002)
50. Chambers, J.M., Mallows, C.L., Stuck, B.W.: A method for simulating stable random variables. Journal of the American Statistical Association **71**, 340–344 (1976)
51. Champernowne, D.G.: The graduation of income distributions. Econometrica **20**, 591–615 (1952)
52. Chavez-Demoulin, V., Embrechts, P., Nešlehová, J.: Quantitative models for operational risk: extremes, dependence and aggregation. Journal of Banking and Finance **30**(9), 2635–2658 (2006)
53. Chernobai, A., Menn, C., Trück, S., Rachev, S.T.: A note on the estimation of the frequency and severity distribution of operational losses. The Mathematical Scientist **30**(2) (2005)
54. Chernobai, A., Rachev, S.T.: Stable modelling of operational risk. In: M.G. Cruz (ed.) Operational Risk Modelling and Analysis. Theory and Practice. Risk Books, London (2004)
55. Chernobai, A.S., Rachev, S.T., Fabozzi, F.J.: Operational Risk: A Guide to Basel II Capital Requirements, Models, and Analysis. Wiley, Hoboken, NJ (2007)
56. Chib, S.: Marginal likelihood from the Gibbs output. Journal of the American Statistical Association **90**(432), 1313–1321 (1995)
57. Clements, A.E., Hurn, A.S., Lindsay, K.A.: Möbius-like mappings and their use in kernel density estimation. Journal of the American Statistical Association **98**, 993–1000 (2003)
58. Clenshaw, C.W., Curtis, A.R.: A method for numerical integration on an automatic computer. Num. Math **2**, 197–205 (1960)
59. Committee of European Banking Supervisors: Guidelines on Operational Risk Mitigation Techniques (December 2009). URL www.c-ebs.org
60. Congdon, P.: Bayesian Statistical Modelling, 2nd edn. Wiley, Chichester, England (2006)
61. Cont, R., Tankov, P.: Financial Modelling With Jump Processes. Chapman & Hall/CRC, Boca Raton, FL (2004)
62. Cope, E.W., Antonini, G., Mignola, G., Ugoccioni, R.: Challenges and pitfalls in measuring operational risk from loss data. The Journal of Operational Risk **4**(4), 3–27 (2009)
63. Cowles, M.K., Carlin, B.P.: Markov chain Monte Carlo convergence diagnostics: a comparative review. Technical report 94-008, Division of Biostatistics, School of Public Health, University of Minessota (1994)
64. Craddock, M., Heath, D., Platen, E.: Numerical inversion of Laplace transforms: a survey of techniques with applications to derivative pricing. Computational Finance **4**(1), 57–81 (2000)
65. Cruz, M.G.: Modeling, Measuring and Hedging Operational Risk. Wiley, Chichester (2002)
66. Cruz, M.G. (ed.): Operational Risk Modelling and Analysis: Theory and Practice. Risk Books, London (2004)
67. Dagpunar, J.S.: An easily implemented generalised inverse Gaussian generator. Communications in Statistics, Simulation and Computation **18**, 703–710 (1989)
68. Dalla Valle, L.: Bayesian copulae distributions, with application to operational risk management. Methodology and Computing in Applied Probability **11**(1), 95–115 (2009)
69. Daul, S., De Giorgi, E., Lindskog, F., McNeil, A.: The grouped t-copula with an application to credit risk. Risk **16**, 73–76 (2003)
70. Davis, E.: Theory vs Reality. OpRisk&Compliance (1 September 2006). URL www.opriskandcompliance.com/public/showPage.html?page=345305
71. Degen, M., Embrechts, P., Lambrigger, D.D.: The quantitative modeling of operational risk: between g-and-h and EVT. ASTIN Bulletin **37**(2), 265–291 (2007)

72. Demarta, S., McNeil, A.: The t copula and related copulas. International Statistical Review **73**, 111–129 (2005)
73. Dempster, A.P.: A generalization of Bayesian inference. Journal of the Royal Statistical Society, Series B **30**, 205–247 (1968)
74. Den Iseger, P.W.: Numerical Laplace inversion using Gaussian quadrature. Probability in the Engineering and Informational Sciences **20**, 1–44 (2006)
75. Denaut, D.: Coherent allocation of risk capital. Journal of Risk **4**(1), 1–34 (2001)
76. Devroye, L.: Non-Uniform Random Variate Generation. Springer, New York, NY (1986)
77. Dutta, K., Perry, J.: A tale of tails: an empirical analysis of loss distribution models for estimating operational risk capital. Federal Reserve Bank of Boston (2006). URL http://www.bos.frb.org/economic/wp/index.htm. Working paper No. 06–13
78. Efron, B.F., Hinkley, D.V.: Assessing the accuracy of the maximum likelihood estimator: observed versus expected Fisher information. Biometrika **65**, 457–487 (1978)
79. Efron, B.F., Tibshirani, R.J.: An Introduction to the Bootstrap. Chapman and Hall, London (1993)
80. Embrechts, P.: A property of the generalized inverse Gaussian distribution with some applications. Journal of Applied Probability **20**, 537–544 (1983)
81. Embrechts, P., Frei, M.: Panjer recursion versus FFT for compound distributions. Mathematical Methods of Operations Research **69**(3), 497–508 (2009)
82. Embrechts, P., Goldie, C., Veraverbeke, N.: Subexponentiality and infinite divisibility. Zeitschrift fur Wahrscheinlichkeitstheorie und Verwandte Gebiete **49**, 335–347 (1979)
83. Embrechts, P., Klüppelberg, C., Mikosch, T.: Modelling Extremal Events for Insurance and Finance. Springer, Berlin (1997). Corrected fourth printing 2003
84. Embrechts, P., Lambrigger, D.D., Wüthrich, M.V.: Multivariate extremes and the aggregation of dependent risks: examples and counter-examples. Extremes **12**(2), 107–127 (2009)
85. Embrechts, P., McNeil, A., Straumann, D.: Correlation and dependence in risk management: properties and pitfalls. In: M. Dempster, H. Moffatt (eds.) Risk Management: Value at Risk and Beyond, pp. 176–223. Cambridge University Press, Cambridge (2002)
86. Embrechts, P., Nešlehová, J., Wüthrich, M.V.: Additivity properties for Value-at-Risk under Archimedean dependence and heavy-tailedness. Insurance: Mathematics and Economics **44**, 164–169 (2009)
87. Embrechts, P., Puccetti, G.: Aggregating risk capital, with an application to operational risk. The Geneva Risk and Insurance Review **31**(2), 71–90 (2006)
88. Embrechts, P., Puccetti, G.: Aggregation operational risk across matrix structured loss data. The Journal of Operational Risk **3**(2), 29–44 (2008)
89. Ergashev, B.: Should risk managers rely on the maximum likelihood estimation method while quantifying operational risk? The Journal of Operational Risk **3**(2), 63–86 (2008)
90. Ergashev, B.: Estimating the lognormal-gamma model of operational risk using the Markov chain Monte Carlo method. The Journal of Operational Risk **4**(1), 35–57 (2009)
91. Fang, H., Fang, K., Kotz, S.: The meta-elliptical distributions with given marginals. Journal of Multivariate Analysis **82**, 1–16 (2002)
92. Federal Reserve System, Office of the Comptroller of the Currency, Office of Thrift Supervision and Federal Deposit Insurance Corporation: Results of the 2004 Loss Data Collection Exercise for Operational Risk (May 2005). URL www.bos.frb.org/bankinfo/qau/papers/pd051205.pdf
93. Ferguson, T.S.: A Course in Large Sample Theory. Chapman and Hall, London (1996)
94. Ferson, S., Kreinovich, V., Ginzburg, L., Myers, D.S., Sentz, K.: Constructing Probability Boxes and Dempster-Shafer Structures. Sandia National Laboratories, Albuquerque, New Mexico 87185 and Livermore, California 94550 (January 2003). SAND report: SAND2002-4015
95. Frachot, A., Moudoulaud, O., Roncalli, T.: Loss distribution approach in practice. In: M. Ong (ed.) The Basel Handbook: A Guide for Financial Practitioners. Risk Books, London (2004)
96. Frachot, A., Roncalli, T.: Mixing internal and external data for managing operational risk. Working paper (2002). Groupe de Recherche Opérationnelle, Crédit Lyonnais, France

97. Frachot, A., Roncalli, T., Salomon, E.: The correlation problem in operational risk. Working paper (2004). Groupe de Recherche Opérationnelle, France
98. Frees, E., Valdez, E.: Understanding relationships using copulas. North American Journal **2**, 1–25 (1998)
99. Gelfand, A., Dey, D.: Bayesian model choice: Asymptotic and exact calculations. Journal of the Royal Statistical Society, series B **56**, 501–514 (1994)
100. Gelman, A., Gilks, W.R., Roberts, G.O.: Weak convergence and optimal scaling of random walks Metropolis algorithm. Annals of Applied Probability **7**, 110–120 (1997)
101. Geman, S., Geman, D.: Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. IEEE Transactions on Pattern Analysis and Machine Intelligence **6**, 721–741 (1984)
102. Gerhold, S., Schmock, U., Warnung, R.: A generalization of Panjer's recursion and numerically stable risk aggregation. Finance and Stochastics **14**(1), 81–128 (2010)
103. Geyer, C.J., Thompson, E.A.: Annealing Markov chain Monte Carlo with applications to ancestral inference. Journal of the American Statistical Association **90**(431), 909–920 (1995)
104. Giacometti, R., Rachev, S.T., Chernobai, A., Bertocchi, M.: Aggregation issues in operational risk. The Journal of Operational Risk **3**(3), 3–23 (2008)
105. Gilks, W.R., Best, N.G., Tan, K.C.: Adaptive rejection Metropolis sampling within Gibbs sampling. Applied Statistics **44**, 455–472 (1995)
106. Gilks, W.R., Richardson, S., Spiegelhalter, D.J.: Markov Chain Monte Carlo in practice. Chapman & Hall, London (1996)
107. Gilks, W.R., Wild, P.: Adaptive rejection sampling for Gibbs sampling. Applied Statistics **41**(2), 337–348 (1992)
108. Glasserman, P.: Monte Carlo Methods in Financial Engineering. Springer, New York, NY (2004)
109. Glasserman, P.: Measuring marginal risk contributions in credit portfolios. Journal Computational Finance **9**(2), 1–41 (2005)
110. Golub, G.H., Welsch, J.H.: Calculation of Gaussian quadrature rules. Mathematics of Computation **23**, 221–230 (1969)
111. Gramacy, R.B., Samworth, R., King, R.: Importance tempering. Technical report, Cambridge University Statistical Laboratory Technical Report Series (2007). Preprint on arXiv:0707.4242
112. Green, P.: Reversible jump MCMC computation and Bayesian model determination. Biometrika **82**, 711–732 (1995)
113. Grønneberg, S., Hjort, N.L.: The copula information criterion. Statistical research report no. 7, Department of Mathematics, University of Oslo (July 2008). ISSN 0806–3842
114. Grubel, R., Hermesmeier, R.: Computation of compound distributions I: aliasing errors and exponential tilting. ASTIN Bulletin **29**(2), 197–214 (1999)
115. Gustafsson, J., Thuring, F.: A suitable parametric model for operational risk applications (February 2008). URL http://ssrn.com/abstract=926309
116. Hastings, W.: Monte Carlo sampling methods using Markov chains and their applications. Biometrika **57**, 97–109 (1970)
117. Haubenstock, M.: The operational risk framework. In: C. Alexander (ed.) Operational Risk: Regulation, Analysis and Management, pp. 241–261. Prentice Hall, New York, NY (2003)
118. Heckman, P.E., Meyers, G.N.: The calculation of aggregate loss distributions from claim severity and claim count distributions. Proceedings of the Casualty Actuarial Society **LXX**, 22–61 (1983)
119. Hess, K.T., Liewald, A., Schmidt, K.D.: An extension of Panjer's recursion. ASTIN Bulletin **32**(2), 283–297 (2002)
120. Hesselager, O.: Recursions for certain bivariate counting distributions and their compound distributions. ASTIN Bulletin **26**(1), 35–52 (1996)
121. Hipp, C.: Speedy convolution algorithms and Panjer recursions for phase-type distributions. Insurance: Mathematics and Economics **38**(1), 176–188 (2006)

122. Hiwatashi, J., Ashida, H.: Advancing Operational Risk Management using Japanese Banking Experiences. Federal Reserve Bank of Chicago (2002)
123. Hoaglin, D.C.: Using quantiles to study shape. In: D.C. Hoaglin, F. Mosteller, J.W. Tukey (eds.) Exploring Data Tables, Trends, and Shapes, pp. 417–460. Wiley, New York, NY (1985a)
124. Hoaglin, D.C.: Summarizing shape numerically: The g-and-h distributions. In: D.C. Hoaglin, F. Mosteller, J.W. Tukey (eds.) Exploring Data Tables, Trends, and Shapes, pp. 461–513. John Wiley & Sons, New York, NY (1985b)
125. Ibragimov, R., Walden, J.: The limits of diversification when losses may be large. Journal of Banking and Finance **31**, 2251–2569 (2007)
126. International Actuarial Association: A Global Framework for Insurer Solvency Assessment (2004). URL www.actuaries.org. A Report by the Insurer Solvency Assessment Working Party of the International Actuarial Association
127. Jeffreys, H.: Theory of Probability, 3rd edn. Oxford University Press, London (1961)
128. Johnson, N.L., Kotz, S., Balakrishnan, N.: Discrete Multivariate Distributions. John Wiley & Sons, New York (1997)
129. Jørgensen, B.: Statistical Properties of the Generalized Inverse Gaussian Distribution. Springer, New York (1982)
130. Kaas, R., Goovaerts, M.J., Dhaene, J., Denuit, M.: Modern Actuarial Risk Theory. Kluwer Academic Publishers, Dordrecht (2001)
131. Kass, R., Raftery A.: Bayes factor. Journal of the American Statistical Association **90**, 773–792 (1995)
132. Kahaner, D., Moler, C., Nash, S.: Numerical Methods and Software. Prentice-Hall, Englewood Cliffs, NJ (1989)
133. Kass, R.E., Carlin, B.P., Gelman, A., Neal, R.M.: Markov chain Monte Carlo in practice: a roundtable discussion. The American Statistician **52**, 93–100 (1998)
134. King, J.L.: Operational Risk: Measurements and Modelling. Wiley, Chichester, England (2001)
135. Klugman, S., Parsa, A.: Fitting bivariate distributions with copulas. Insurance: Mathematics and Economics **24**, 139–148 (1999)
136. Klugman, S.A., Panjer, H.H., Willmot, G.E.: Loss Models: From Data to Decisions. Wiley, New York, NY (1998)
137. Kogon, S.M., William, B.W.: Characteristic function based estimation of stable distribution parameters. In: R.J. Adler, R.E. Feldman, M.S. Taqqu (eds.) A Practical Guide to Heavy Tails: Statistical Techniques and Applications. Birkhäuser, Boston, MA (1998)
138. Kolmogorov, A.N.: Grundbegriffe der Wahrscheinlichkeistrechung. Ergebnisse der Mathematik, Springer, Berlin (1933)
139. Kronrod, A.S.: Nodes and weights of quadrature formulas. Sixteen-place tables. New York: Consultants Bureau Authorized translation from Russian Doklady Akad. Nauk SSSR **154**, 283–286 (1965)
140. Lam, J.: Enterprise Risk Management: From Incentives to Controls. Wiley, Hoboken, NJ (2003)
141. Lambrigger, D.D., Shevchenko, P.V., Wüthrich, M.V.: The quantification of operational risk using internal data, relevant external data and expert opinions. The Journal of Operational Risk **2**(3), 3–27 (2007)
142. Lavin, M., Scherrish, M.: Bayes factors: what they are and what they are not. The American Statistician **53**, 119–122 (1999)
143. Lehmann, E.L.: Theory of Point Estimation. Wiley, New York, NY (1983)
144. Lehmann, E.L., Casella, G.: Theory of Point Estimation, 2nd edn. Springer, New York, NY (1998)
145. Lindskog, F., McNeil, A.: Common Poisson shock models: application to insurance and credit risk modelling. ASTIN Bulletin **33**, 209–238 (2003)
146. Litterman, R.: Hot spots^{TM} and hedges. The Journal of Portfolio Management **22**, 52–75 (1996)

147. Luo, X., Shevchenko, P.V.: Computing tails of compound distributions using direct numerical integration. The Journal of Computational Finance **13**(2), 73–111 (2009)
148. Luo, X., Shevchenko, P.V.: The t copula with multiple parameters of degrees of freedom: bivariate characteristics and application to risk management. Quantitative Finance **10**(9), 1039–1054 (2010)
149. Luo, X., Shevchenko, P.V.: A short tale of long tail integration. Numerical Algorithms (2010). DOI: 10.1007/s11075-010-9406-9
150. Luo, X., Shevchenko, P.V.: Bayesian model choice of grouped t-copula (2011). To appear in Methodology and Computing in Applied Probability
151. Luo, X., Shevchenko, P.V., Donnelly, J.: Addressing impact of truncation and parameter uncertainty on operational risk estimates. The Journal of Operational Risk **2**(4), 3–26 (2007)
152. MacEachern, S.N., Berliner, L.M.: Subsampling the Gibbs sampler. The American Statistician **48**, 188–190 (1994)
153. Marinari, E., Parisi, G.: Simulated tempering: a new Monte Carlo scheme. Europhysics Letters **19**(6), 451–458 (1992)
154. Marshall, C.L.: Measuring and Managing Operational Risks in Financial Institutions. Wiley, Singapore (2001)
155. Martinez, J., Iglewicz, B.: Some properties of the tukey g and h family of distributions. Communications in Statistics – Theory and Methods **13**(3), 353–369 (1984)
156. McDonald, J.B., Xu, Y.J.: A generalization of the beta distribution with applications. The Journal of Econometrics **66**, 133–152 (1995)
157. McNeil, A.J., Frey, R., Embrechts, P.: Quantitative Risk Management: Concepts, Techniques and Tools. Princeton University Press, Princeton, NJ (2005)
158. Melchiori, M.: Tools for sampling multivariate Archimedian copulas. Preprint, www.yieldcurve.com
159. Meng, X., Wong, W.: Simulating ratios of normalizing constants via a simple identity. Statistical Sinica **6**, 831–860 (1996)
160. Menn, C., Rachev, S.T.: Calibrated FFT-based density approximation for alpha stable distributions. Computational Statistics and data Analysis **50**, 1891–1904 (2006)
161. Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., Teller, E.: Equations of state calculations by fast computing machines. Journal of Chemical Physics **21**, 1087–1091 (1953)
162. Miazhynskaia, T., Dorffner, G.: A comparison of Bayesian model selection based on MCMC with an application to GARCH-type models. Statistical Papers **47**, 525–549 (2006)
163. Mignola, G., Ugoccioni, R.: Effect of a data collection threshold in the loss distribution approach. The Journal of Operational Risk **1**(4), 35–47 (2006)
164. Mignola, G., Ugoccioni, R.: Tests for extreme value theory. Operational Risk&Compliance pp. 32–35 (October 2005)
165. Mira, A., Tierney, L.: Efficiency and convergence properties of slice samplers. Scandinavian Journal of Statistics **29**(1), 1–12 (2002)
166. Moscadelli, M.: The modelling of operational risk: experiences with the analysis of the data collected by the Basel Committee. Bank of Italy (2004). Working paper No. 517
167. Muermann, A., Oktem, U.: The near-miss management of operational risk. The Journal of Risk Finance **4**(1), 25–36 (2002)
168. Neal, R.M.: Probabilistic inference using Markov chain samplers. Technical report, Department of Computer Science, University of Toronto (1993)
169. Neal, R.M.: Sampling from multimodal distributions using tempered transitions. Statistics and Computing **6**, 353–366 (1996)
170. Neal, R.M.: Slice sampling (with discussions). Annals of Statistics **31**, 705–767 (2003)
171. Neil, M., Fenton, N.E., Tailor, M.: Using Bayesian networks to model expected and unexpected operational losses. Risk Analysis **25**(4), 963–972 (2005)
172. Neil, M., Häger, D., Andersen, L.B.: Modeling operational risk in financial institutions using hybrid dynamic Bayesian networks. Journal of Operational Risk **4**(1), 3–33 (2009)

173. Neil, M., Malcolm, B., Shaw, R.: Modelling an air traffic control environment using Bayesian belief networks. Ottawa, Ontario, Canada (August 2003). 21st International System Safety Conference
174. Nešlehová, J., Embrechts, P., Chavez-Demoulin, V.: Infinite mean models and the LDA for operational risk. Journal of Operational Risk **1**(1), 3–25 (2006)
175. Newton, M., Raftery, A.: Approximate Bayesian inference by the weighted likelihood bootstrap. Journal of the Royal Statistical Society, series B **56**, 1–48 (1994)
176. Nolan, J.: Stable Distributions – Models for Heavy Tailed Data. Birkhauser, Boston, MA (2007)
177. Nolan, J.P.: Numerical calculation of stable densities and distribution functions. Communications in Statistics – Stochastic Models **13**, 759–774 (1997)
178. O'Hagan, A., Buck, C.E., Daneshkhah, A., Eiser, J.R., Garthwaite, P.H., Jenkinson, D., Oakley, J.E., Rakov, T.: Uncertain Judgements: Eliciting Expert's Probabilities. Wiley, Chichester (2006)
179. Panjer, H., Willmot, G.: Insurance Risk Models. Society of Actuaries, Chicago, IL (1992)
180. Panjer, H.H.: Recursive evaluation of a family of compound distribution. ASTIN Bulletin **12**(1), 22–26 (1981)
181. Panjer, H.H.: Operational Risks: Modeling Analytics. Wiley, New York, NY (2006)
182. Panjer, H.H., Wang, S.: On the stability of recursive formulas. ASTIN Bulletin **23**(2), 227–258 (1993)
183. Panjer, H.H., Willmot, G.E.: Computational aspects of recursive evaluation of compound distributions. Insurance: Mathematics and Economics **5**, 113–116 (1986)
184. Peters, G.W., Byrnes, A.D., Shevchenko, P.V.: Impact of insurance for operational risk: Is it worthwhile to insure or be insured for severe losses? Insurance: Mathematics and Economics (2010), doi:10.1016/j.insmatheco.2010.12.001
185. Peters, G.W., Johansen, A.M., Doucet, A.: Simulation of the annual loss distribution in operational risk via Panjer recursions and Volterra integral equations for value-at-risk and expected shortfall estimation. The Journal of Operational Risk **2**(3), 29–58 (2007)
186. Peters, G.W., Shevchenko, P.V., Wüthrich, M.V.: Model uncertainty in claims reserving within Tweedie's compound Poisson models. ASTIN Bulletin **39**(1), 1–33 (2009)
187. Peters, G.W., Shevchenko, P.V., Wüthrich, M.V.: Dynamic operational risk: modeling dependence and combining different data sources of information. The Journal of Operational Risk **4**(2), 69–104 (2009)
188. Peters, G.W., Sisson, S.A.: Bayesian inference, Monte Carlo sampling and operational risk. The Journal of Operational Risk **1**(3), 27–50 (2006)
189. Peters, G.W., Sisson, S.A., Fan, Y.: Simulation for Bayesian models constructed with alpha stable distributions utilising approximate Bayesian computation (2009)
190. Peters, G.W., Wüthrich, M.V., Shevchenko, P.V.: Chain ladder method: Bayesian bootstrap versus classical bootstrap. Insurance: Mathematics and Economics **47**(1), 36–51 (2010)
191. Peters, J-P., Hübner, G.: Modeling operational risk based on multiple experts' opinions. In: G.N. Gregoriou (ed.) Operational Risk Toward Basel III: Best Practices and Issues in Modeling, Management, and Regulation. Wiley, New York (2009)
192. Piessens, R., Doncker-Kapenga, E.D., Überhuber, C.W., Kahaner, D.K.: QUADPACK – a Subroutine Package for Automatic Integration. Springer, New York, NY (1983)
193. Planning and Coordination Bureau, Financial Service Agency, Financial Systems and Bank Examination Department, Bank of Japan: Results of the 2007 Operational Risk Data Collection Exercise (August 2007). URL www.boj.or.jp/en/type/ronbun/ron/research07/ron0709a.htm
194. Powojowski, M.R., Reynolds, D., Tuenter, H.J.H.: Dependent events and operational risk. ALGO Research Quarterly **5**(2), 65–73 (2002)
195. Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P.: Numerical Recipes in C. Cambridge University Press, New York, NY (2002)
196. Pugachev, V.S.: Theory of Random Functions and its applications to control problems, 1st edn. Pergamon Press, London (1965)

197. Rachev, S.T., Mittnik, S.: Stable Paretian Models in Finance. Wiley, New York, NY (2000)
198. Rayner, G.D., MacGillivray, H.L.: Numerical maximum likelihood estimation for the g-and-h and generalized g-and-h distributions. Statistics and Computing **12**, 57–75 (2002)
199. Ripley, B.D.: Stochastic Simulation. Wiley, New York, NY (1987)
200. Robert, C.P.: The Bayesian Choice. Springer, New York, NY (2001)
201. Robert, C.P., Casella, G.: Monte Carlo Statistical Methods, 2nd edn. Springer Texts in Statistics, New York, NY (2004)
202. Roberts, G.O., Rosenthal, J.S.: Optimal scaling for various Metropolis-Hastings algorithms. Statistical Science **16**, 351–367 (2001)
203. Robertson, J.: The computation of aggregate loss distributions. Proceedings of the Casuality Actuarial Society **79**, 57–133 (1992)
204. Rosenthal, J.S.: AMCMC: An R interface for adaptive MCMC. Computational Statistics and Data Analysis **51**(12), 5467–5470 (2007)
205. Rosenthal, J.S.: Optimal proposal distributions and adaptive mcmc. In: A. Gelman, J. G., X.L. Meng, S. Brooks (eds.) Handbook of Markov chain Monte Carlo: Methods and Applications. Chapman & Hall /CRC Press, Florida, FL (2009)
206. Rytgaard, M.: Estimation in Pareto distribution. ASTIN Bulletin **20**, 201–216 (1990)
207. Sandström, A.: Solvency: Models, Assessment and Regulation. Chapman & Hall/CRC, Boca Raton, FL (2006)
208. Savage, L.J.: The subjective basis of statistical practice. Department of Statistics, University of Michigan, Ann Arbor (1961). Technical report
209. Schwarz, G.: Estimation the dimension of a model. Annals of Statistics **6**, 461–464 (1978)
210. Schmock, U.: Estimating the value of the Wincat coupons of the Winterthur insurance convertible bond: a study of the model risk. ASTIN Bulletin **29**(1), 101–163 (1999)
211. Seal, H.L.: Numerical inversion of characteristic functions. Scandinavian Actuarial Journal pp. 48–53 (1977)
212. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton, NJ (1976)
213. Sharpe, W.F.: Capital asset prices: a theory of market equilibrium under conditions of risk. Journal of Finance **19**, 425–442 (1964)
214. Shephard, N.G.: From characteristic function to distribution function: a simple framework for the theory. Econometric Theory **7**, 519–529 (1991)
215. Shevchenko, P.V.: Estimation of operational risk capital charge under parameter uncertainty. The Journal of Operational Risk **3**(1), 51–63 (2008)
216. Shevchenko, P.V.: Implementing loss distribution approach for operational risk. Applied Stochastic Models in Business and Industry **26**(3), 277–307 (2010)
217. Shevchenko, P.V., Temnov, G.: Modeling operational risk data reported above a time-varying threshold. The Journal of Operational Risk **4**(2), 19–42 (2009)
218. Shevchenko, P.V., Wüthrich, M.V.: The structural modeling of operational risk via Bayesian inference: combining loss data with expert opinions. The Journal of Operational Risk **1**(3), 3–26 (2006)
219. Sidi, A.: Extrapolation methods for oscillatory infinite integrals. Journal of the Institute of Mathematics and Its Applications **26**, 1–20 (1980)
220. Sidi, A.: A user friendly extrapolation method for oscillatory infinite integrals. Mathematics of Computation **51**, 249–266 (1988)
221. Smith, R.L.: Estimating tails of probability distributions. Annals of Statistics **15**, 1174–1207 (1987)
222. Steinhoff, C., Baule, R.: How to validate op risk distributions. OpRisk&Compliance pp. 36–39 (August 2006)
223. Stoer, J., Bulirsch, R.: Introduction to Numerical Analysis, 3rd edn. Springer, New York, NY (2002)
224. Stuart, A., Ord, J.K.: Kendall's Advanced Theory of Statistics: Volume 1, Distribution Theory, Sixth Edition. Edward Arnold, London/Melbourne/Auckland (1994)

225. Stuart, A., Ord, J.K., Arnold, S.: Advanced Theory of Statistics, volume 2A: Classical Inference and the Linear Models, 6th edn. Oxford University Press, London (1999)
226. Sundt, B.: On some extensions of Panjer's class of counting distributions. ASTIN Bulletin **22**(1), 61–80 (1992)
227. Sundt, B.: On multivariate Panjer recursions. ASTIN Bulletin **29**(1), 29–45 (1999)
228. Sundt, B., Jewell, W.S.: Further results on recursive evaluation of compound distributions. ASTIN Bulletin **12**(1), 27–39 (1981)
229. Sundt, B., Vernic, R.: Recursions for Convolutions and Compound Distributions with Insurance Applications. Springer, Berlin (2009)
230. Swiss Financial Market Supervisory Authority (FINMA), Bern, Switzerland: Swiss Solvency Test, Technical Document (2006)
231. Szegö, G.: Orthogonal Polynomials, 4th edn. American Mathematical Society, Providence, RI (1975)
232. Tasche, D.: Risk contributions and performance measurement (1999). URL http://www-m4.ma.tum.de/pers/tasche. Preprint, Department of Mathematics, TU München
233. Tasche, D.: Euler Allocation: Theory and Practice (2008). Preprint arXiv:0708.2542v2 available on http://arxiv.org
234. Tavaré, S., Marjoram, P., Molitor, J., Plagnol, V.: Markov chain Monte Carlo without likelihoods. Proceedings of the National Academy of Science, USA **100**, 15,324–15,328 (2003)
235. Tukey, J.W.: Exploratory Data Analysis. Addison-Wesley, Boston, MA (1977)
236. Vernic, R.: Recursive evaluation of some bivariate compound distributions. ASTIN Bulletin **29**(2), 315–325 (1999)
237. Waller, L.A., Turnbull, B.G., Hardin, J.M.: Obtaining distribution functions by numerical inversion of characteristic functions with applications. The American Statistician **49**(4), 346–350 (1995)
238. Wasserman, L.: Bayesian model selection and model averaging. Technical report, Statistics Department, Carnegie Mellon University (1997)
239. Wüthrich, M.V.: Premium liability risks: modelling small claims. Bulletin of the Swiss Association of Actuaries **1**, 27–38 (2006)
240. Wüthrich, M.V., Merz, M.: Stochastic Claims Reserving Methods in Insurance. Wiley, Chichester, England (2008)
241. Wynn, P.: On a device for computing the $e_m(s_n)$ tranformation. Mathematical Tables and Other Aids to Computation **10**, 91–96 (1956)
242. Yamai, Y., Yoshiba, T.: Comparative analyses of expected shortfall and Value-at-Risk: Their estimation error, decomposition, and optimization. Monetary and Economic Studies pp. 87–121 (January 2002)

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